FINITE ELEMENT METHOD IN THE FLUID FLOW AND HEAT TRANSFER PROBLEMS

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ABSTRACT

Engineering problems of heat transfer and fluid flow involve one or more governing equations with some boundary conditions over a domain. In general, the domain of the problem is not simple but often complex and nonuniform so it is not possible to find an exact solution every time. A finite element method is a powerful tool for numerically analyzing problems in these areas because of the ability to accurately discretize domains of any size and shape to use a mesh of finite elements. This paper reviews the applications of finite element approaches in heat transfer and fluid flow, and highlights some recent advances in this method.

Keywords: *Finite Element Method, Fluid Flow, Heat Transfer, Mesh Generation.*

The introduction: The period prior to the introduction of computers, engineering problems often demanded a large amount of time to derive analytic or exact solutions. Although these solutions often provided excellent insight into the behavior of some systems, it could be derived only for limited problems. Since the late 1940s, the availability of digital computers has led to an unquestionable explosion in the development and use of numerical methods. These techniques have great capabilities to solve complex problems and handle large systems of equations, nonlinear behavior, and complicated geometries that are often difficult or impossible to solve analytically.

For example, the governing equation of the fundamental two-dimensional heat conduction problem is:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad (1)$$

where $u(x, y)$ is the temperature distribution in the Cartesian coordinates x, y, and is defined in a rectangular region $0 < x < a$, $0 < y < b$, together with the boundary conditions:

$$u(0, y) = 0 \text{ and } u(a, y) = 0 \quad \forall 0 \le y \le b$$

$$u(x,0) = 0 \text{ and } u(x,b) = u_b \quad 0 \le x \le a \quad (2)$$

This equation has an analytical solution [1]:

$$u(x, y) = \frac{4u_0}{\pi} \sum_{k=1}^{\infty} \frac{1}{(2k+1)} \frac{\sinh \frac{(2k+1)}{a} \pi y}{\sinh \frac{(2k+1)}{a} \pi b} \sinh \frac{(2k+1)}{a} \pi x \quad (3)$$

This expression is not simple, and still we need numerical procedure to evaluate this. It is desirable to recast the problem by considering various forms of discretization. The typical term of a differential can be converted to approximate discrete expression by using a numerical scheme. The discretized form of the problem only requires the solution to be satisfied at a finite number of points in the region; and in the remainder of the region, appropriate interpolations may be used. Thus, the problem is reduced to a purely algebraic form involving only the basic arithmetic operations, which could in turn be solved by numerical methods. We have many general discretization methods namely, the Finite difference method, Finite Element Method, etc. The Finite Difference Method is the simplest method to apply, but particularly on the uniform grid, and requires high degree mesh regularities. One common numerical technique in engineering analysis is the finite element method for solving initial and boundary value problems.

History: The modern use of finite elements started in the field of structural engineering. The advent of jet engine in the 1940s and the resulting changes in aircraft speeds had led to the change from unswept to swept wind designs. The first attempt was by Hrennikoff [1] who developed analogy between actual discrete elements and the corresponding portions of a continuous solid, and it was adapted to aircraft structural design. Based on Displacement Assumptions, Turner et al. [4] introduced the element stiffness matrix for a triangular element, and together with the direct stiffness method, described the method for assembling the elements. Clough [7] introduced the term 'finite element' in a paper describing the applications in plane elasticity. Works on the solution of non linearity problems had become more prominent. Incremental technique to solve geometrical non linearity problems was initiated by Turner et al. [4], and stability problems were analysed by Martin [3]. Material nonlinearity problems, such as plasticity and viscoelasticity, were discussed by Gallagher et al. [8] and Zienkiewicz et al. [5] respectively. Melosh [8] utilized the principle of minimum potential energy and provided the first convergence proof in the engineering literature. This led to the use of variational principle that extended the use of FEM in many new areas. Zienkiewicz and Cheung [6] examined the solution of Possion's equation, and Wilson and Nickell [2] considered the transient heat conduction problems. The method also found applications in the field of biomedical engineering, where geometric and material non linearity would be involved. This problem was first investigated by Gould et al. [10].

FINITE ELEMENT METHOD:

The fundamental idea of the FEM is to discretise the domain into several sub domains, or finite elements. These elements can be irregular and possess different properties so that they form a basis to discretise complex structures, or structures with mixed material properties. Further, they can accurately model the domain boundary regardless of its shape. To establish a 'general purpose' method for solving problems in heat transfer and fluid flow, consider the system of differential equations:

Au = f in Ù (4)
with the boundary conditions:
Bu = t in Γ (5)

where A is a system of governing equations defined in the domain Ù, B is a system of some boundary functions defined in the boundary Γ, and f, t are some functions. This system governs many applications in the engineering field. To find a solution to this system, apply the weighted residual method and yield:

$$\int_{\Omega} W_j(Au - f)d\Omega + \oint_{\Gamma} W_j(B\bar{u} - t)d\Gamma = 0 \quad (6)$$

where *Wj(j = 1,...,n)* are weighting functions and $\bar{u}$ is an approximation to the unknown *u*:

$$u = \bar{u} = \sum_{j=1}^{n} N_j u_j \quad (7)$$

in which N_j are some basis functions and u_j are the nodal values of the unknown.

Substituting equation (7) into equation (6), a system of equations can be obtained:
$Ku = f$ (8)

where K is a square matrix, and u, f are some vectors.

The Galerkin version of FEM (GFEM) is defined when the weighting function in equation (6) is: (9)
$W_j = N_j$ *(9)*

This method leads to minimum errors and preserves the symmetry of matrix K, and it is the most frequently used version of FEM. Sometimes this method is also called the Bubnov-Galerkin methods (BGFEM). In recent years we have some other versions of FEM like, Petrov-Galerkin finite element method (PGFEM), the finite volume method (FVM) etc.

APPLICATIONS TO HEAT TRANSFER AND FLUID FLOW

1. Hybrid Schemes for Solving Nonlinear Convection-Diffusion and Compressible

Viscous Flow Problems-The viscosity and heat conduction coefficients of gases are small, so that viscous dissipative terms are often considered as perturbations in the inviscid Euler system. It implies that an effective numerical method for solving inviscid flow must be considered. A hybrid FVM and FEM scheme is proposed to solve nonlinear convection-diffusion problems and compressible viscous flow using a general class of cell-centred flux vector splitting FVM discretization of inviscid terms together with FEM discretization of viscous terms over a triangular grid.

2. Spatially Periodic Flows in Irregular Domains

Based on the relative orientation of the modules, two types of periodicity are considered translational and rotational. When the geometry of the flow problem is complex, periodic boundary fitted grids are often used over a typical module to predict such flows. Finite volume non staggered grid methods are often used to discretise the momentum and continuity equations in fluid flow.

3. Acoustic Fluid-structure Interaction - Problems In this method, the pure displacement-based formulation is replaced by a displacement/pressure (u/p) formulation via a variational indicator. The standard Galerkin finite element discretization procedure is applied to give the matrix equations of the u/p formulation.

Conclusion:

FEM offers enormous flexibility in the treatment of nonlinearities, inhomogeneities and anisotropy. The objective of this paper was to identify some trends in FEM and their relation to research in engineering. It is hoped that works from different disciplines, whose common interest is finite element methods, can promote wider awareness throughout the finite element community of the latest developments in engineering and mathematics.

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